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1994 J. Phys. A: Math. Gen. 27 6157

(http://iopscience.iop.org/0305-4470/27/18/024)

J. Phys. A: Math. Gen. 27 (1994) 6157–6166. Printed in the UK

Stability estimate in scattering theory and its application to mesoscopic systems and quantum chaos

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Received 5 April 1994, in final form 20 June 1994

Abstract. We consider the scattering of a free quantum particle on a singular potential with rather arbitrary support boundary geometry. In the classical limit $\hbar = 0$, this problem reduces to the well known problem of chaotic scattering. The universal estimates for the stability of the scattering amplitudes are derived. The application of the obtained results to the mesoscopic systems and quantum chaos are discussed. We also discuss a possibility of experimental verification of the obtained results.

1. Introduction

Recently, much attention has been paid to the theoretical and experimental investigations of the scattering of a free quantum particle off obstacles with rather complicated boundary forms. Studies of the scattering processes of an electron with mesoscopic resonant tunnelling structures, when quantum effects and the geometry of the scattering potential are important [1–13], are of special interest. Usually, these quantum systems are non-integrable, which means that the number of degrees of freedom exceeds the number of global independent integrals of motion. In this case, one cannot separate variables in the Schrödinger equation or the corresponding Newtonian equations. Treated classically, these systems exhibit dynamical chaos, i.e. strong (exponential) instability of motion under a small variation of parameters (such as the energy of an incident wave, form of the potential, etc). This is why one of the main problems in studying such systems is to determine the role and contribution of fluctuations and correlations in the scattering amplitudes and cross sections [14–22].

The problem of fluctuations in the scattering amplitudes and cross sections of elastic (and inelastic) collision processes is well known and has a long history (see [23–28] and references therein). In elastic scattering, the fluctuations of the scattering amplitudes can appear because of a high sensitivity to the details of the scattering: the parameters of the incident wave and the geometry of the scatter potential. At the same time, the coherent effects (correlations) are also present in the scattering processes in some region of parameters [21–24, 29]. Thus, the problem arises: how does one separate and describe the random and

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coherent effects in the scatttering processes, and how does one measure their contribution in experiments?

The first theoretical investigations into statistical properties (fluctuations) of scattering amplitudes and cross sections were presented in [23–28] (Ericson fluctuations). According to [23, 24], the main reasons why the scattering amplitudes become random are as follows. Let an incident wave have a wavelength $\lambda = 2\pi/k$ much smaller than the characteristic dimension L of the region D where the scattering potential $q(x)$ (see below) is located: $kL \gg 1$ and $x = (x_1, \dots, x_n)$. Before escaping the region D, the incident wave can be reflected many times off the boundaries Γ_j of the potential support $q(x)$. In this case, a wave similar to a standing wave appears in the system. These 'quasi-standing' (or quasy-stationary) waves can be associated with the resonances in the scattering amplitude. These resonances are observed in various experiments and, recently, in mesoscopic systems applications which are very promising for the future (see, for example, [30–32]). Each nth resonance is characterized by two main parameters: the energy E_n and the width Γ_n [33]. There is usually one more important parameter which characterizes the spacing between the neighbouring resonances: ΔE_n. Because the process of scattering is completely defined, the scattering amplitudes should be reproducible in different experiments, providing that all conditions remain identical. However, as was mentioned above, under the condition $kL \gg 1$, the number of reflections of the incident wave in the region D can be very large (in [23, 24], the following inequality is also assumed to be satisfied: $\Gamma_n/\Delta E_n \gg 1$, which is called the regime of overlapping levels). Then, a small variation of parameters can completely change the 'trajectory' of the wave and, consequently, the phase of the scattering amplitude. These ideas were developed in [23, 24, 26] on the basis of the statistical approach [34].

Recently, the problem of scattering-amplitude fluctuations has attracted additional interest in connection with the so-called 'chaotic (irregular) scattering' (CS) in chemical reactions, particle physics, mesoscopic systems and other areas of physics [14–22, 35–37]. The investigations into CS can be divided conventionally into three groups: (i) classical CS (CCS); (ii) semiclassical CS (SCS); and (iii) quantum CS (QCS). The basic ideas are associated with CCS, since only in this case does the 'real dynamical chaos' occur. The investigations into CCS were stimulated by the significant progress achieved recently in the study of dynamical chaos in classically bounded Hamiltonian systems [38–41]. The classical phase space in this case can be very complicated and each of the trajectories belongs to one of the following three classes: (a) stable periodical trajectories; (b) unstable periodical trajectories; and (c) chaotic (unperiodical) trajectories. Dynamical chaos in bounded systems is stationary in the sense that it does not disappear at large times ($T \to \infty$). The systems where CCS takes place are unbounded and additional trajectories appear: (d) unbounded trajectories. For these trajectories chaos can only be transient.

A special interest in the scattering problem is represented by singular potentials $q(x)$ which satisfy the following property: $q(x) = \infty$ when $x \in D$ and $q(x) = 0$ when $x \notin D$, where D is some compact (located region) in R^n. In the case of a singular potential $q(x)$ considered below, the (a) trajectories can be absent (see, for example, [15]) and the (b) and (c) trajectories represent a so-called 'repeller' Ω_R [15]. For the (d) trajectories, this repeller leads to 'transient chaos' which was previously investigated in various bound conservative and dissipative systems (see, for example, [42–44]). In this sense, a singular scattering potential leads to the most chaotic classical 'repeller' and to the biggest fluctuation level in the scattering amplitude.

The main achievements in CCS are associated with the understanding of the following facts: (i) although, in CCS, a direct contribution in the cross section is connected with the (d) trajectories, the influence of the repeller Ω_R-bounded (trapped) trajectories on the scattering and fluctuation process plays a very important role; (ii) CCS is a general phenomenon rather then an exception. (In some special cases of a singular potential [45], the set Ω_R can consist of only one unstable periodic trajectory.) Usually, for singular potentials, the repeller Ω_R is a Cantor set with a fractal structure (see, for example, [15] where an elastic scattering on three hard discs (3HD) was considered) and is characterized by several quantities, such as the Hausdorff dimension D_H, Lyapunov exponents λ_i, the Kolmogorov–Sinai entropy per unit time h_{KS}, the escape rate γ and other quantities (see [15] and references therein). There are some relations between these parameters, for example (see [15])

$$\gamma = \sum_{\lambda_i > 0} \lambda_i - h_{KS}. \tag{1.1}$$

The escape rate γ is a classical equivalent of the resonance width Γ: $\gamma \sim \Gamma/\hbar$ [15]. So, relation (1.1) shows a fundamental property of CCS: when the repeller Ω_R is chaotic ($h_{KS} > 0$), the escape rate (and the resonance width Γ) is decreasing. Also, in this case, large fluctuations appear in the quantities which characterize the process of CCS, for example, in the time delay function [15, 20, 22].

When one investigates SCS and QCS, the main problem is: what are the 'fingerprints' of the classical chaos on the quantum scattering? The problem of QCS was considered for the first time in [14], where the elastic scattering was studied on a two-dimensional surface of constant negative curvature. According to [14], the scattering phase shift as a function of the momentum is given by the phase angle of the Riemann's zeta function and displays a very complicated (chaotic) behaviour (see, for details, [14, 21, 22]). In [16], SCS was studied in the 3HD system using the analysis based on the Gutzwiller trace formula [46]. This trace formula is valid when all periodic orbits of the repeller Ω_R are unstable and isolated. Both these conditions can be satisfied for the singular potential $q(x, t \to \infty)$ considered in sections 2 and 3, including the particular case of a singular potential of the 3HD system considered in [15–18].

The quantum analysis presented in [21, 22] shows that in QCS, the statistical properties of the fluctuations in the cross section can be described by the theory of random matrix ensembles [41]. Different aspects on the problem of fluctuations in SCS and QCS are discussed in [14, 16–28, 35–37].

At the same time, much less is known about the contribution and characteristic properties of the correlations (coherent component) in CS. As was pointed out in [23, 24], a significant level of correlations in the cross section should be expected when, for example, the energy change δE of the incident wave in (2) is small compared with the resonance width Γ ($\Gamma/\delta E > 1$). According to [23, 24], in this case, essentially the same states are excited and the scattering amplitudes are changed insignificantly. The existence of correlations in QCS was also discussed in [21, 22] for some quasi-1D periodical potential (in [22], an experiment is discussed in connection with the correlations in the chaotic scattering). It was shown in [21, 22] that the energy correlations for the matrix elements of the S-matrix exist and exhibit themselves when $\Gamma/\delta E > 1$, in agreement with the Ericson hypothesis [33, 34].

The same problem of the contribution of correlation and fluctuation effects arises when calculating a transition probability of an injected electron transmitted through mesoscopic devices such as the double-barrier resonance tunnelling structure (DBRTS), quantum dots and others [8–13, 30–32]. In this case, a transmitted electron 'feels' the boundary of the scattering potential $q(x)$ and the transmission amplitude can vary significantly depending on the small variation of the sample's form. As discussed above, we again come, in this case, to the problem of CS. So, both these problems, QCS and the scattering problem in mesoscopic systems, are strongly connected. The correlation properties of the transmission amplitude in the mesoscopic system, at large values of Γ, were observed experimentally in [32].

In this situation, it is important to consider the problem of contribution of correlations and fluctuations to QCS for a rather general class of potentials $q(x)$. Although, in this case, only general statements about the scattering amplitude can be made, such an approach has an obvious advantage: these general statements can be applied to a wide variety of systems. From this point of view, singular potentials are of particular interest because they may produce the 'repeller' Ω_R, which can be 'extremely chaotic' and the level of fluctuations can significantly increase. There are no results allowing us to estimate the contribution of fluctuations and correlations in the scattering amplitude in this case. Our main result, discussed below, is that even in the case of strongly singular potentials, some universal correlations exist in the scattering amplitude.

In this paper, we consider a scattering problem for a free quantum particle scattered by a bounded obstacle with rather arbitrary boundary shapes. The boundary may consist of several connected components. As was already mentioned, a similar situation occurs in ballistic scattering processes in the mesoscopic systems widely considered nowadays. The results obtained in this paper can be formulated as follows. It is shown that there exists a region of parameters where a small variation of rather arbitrary strongly singular potential (note that the variation of the total energy is infinite for the potentials we consider) leads only to small variations of the scattering amplitudes. This parameter region can be defined as a region of strong correlations. These correlations are universal in the sense that they do not depend on the concrete structure of the resonances in the scattering amplitude in a particular system under consideration. Because the results we discuss in this paper are given in the form of exact statements, we outline their proofs.

The paper is organized as follows. In section 2, we present a stability estimate for the scattering amplitudes for a rather wide class of potentials. In section 3, a proof of the stability of the scattering amplitudes is given for a singular potential. Discussion of the results is given in section 4.

2. Stability estimate for the scattering amplitude

In this section, we prove that small variations in the potential lead to small perturbations in the scattering amplitude for a class of strongly singular potentials which can take infinite values on sets of positive measure. The notion of small variations will be specified.

(1) Let $D = \bigcup_{j=1}^{J} D_j$, $\Gamma := \partial D = \bigcup_{j=1}^{J} \Gamma_j$, where $D_j \subset R^n$ is a bounded domain with a $C^{2,\nu}$, $0 < \nu \leqslant 1$, boundary Γ_j. This means that, in the local coordinates, the equation of $\Gamma_j := \partial D_j$ is $x_n = \phi(x')$, $x' := (x_1, x_2, \ldots, x_{n-1})$, $\phi \in C^{2,\nu}$, $\|\phi\|_{C^{2,\nu}} \leqslant \Phi_\nu$.

Assume $D \subset B_a := \{x : |x| \leqslant a\}$ and $D_j \bigcap D_i = \emptyset$ if $i \neq j$, $J < \infty$. Define $u_0 := \exp(\mathrm{i}k\alpha \cdot x)$ and

$$q(x;t) := t\chi_D(x) \qquad \chi_D(x) := \begin{cases} 1 & \text{in } D \\ 0 & \text{in } D' := R^n \setminus D \end{cases}$$

where parameter $t \in [1, \infty]$. For definiteness, we take only $n = 3$ in what follows. Consider the scattering problem

$$[\nabla^2 + k^2 - q(x;t)]u = 0 \qquad \text{in } R^3 \tag{2.1}$$

$$u = \exp(ik\alpha \cdot x) + A^{(t)}(\alpha', \alpha, k)\frac{\exp(ikr)}{r} + o\left(\frac{1}{r}\right) \qquad r := |x| \to \infty \qquad \frac{x}{|x|} := \alpha'. \tag{2.2}$$

The scattering solution $u(x, \alpha, k; t) := u(t)$ is uniquely defined as the solution of (2.1) and (2.2). It was proved in [47–49] that

$$|u(t) - u_\Gamma| \to 0 \qquad \text{as } t \to +\infty \tag{2.3}$$

where u_Γ is the scattering solution to the obstacle scattering problem

$$(\nabla^2 + k^2)u_\Gamma = 0 \text{ in } D' \qquad u_\Gamma = 0 \text{ on } \Gamma \tag{2.4}$$

$$u_\Gamma = u_0 + A_\Gamma(\alpha', \alpha, k)\frac{\exp(ikr)}{r} + o\left(\frac{1}{r}\right) \qquad r = |x| \to \infty \qquad \alpha' := \frac{x}{r}. \tag{2.5}$$

The relation (2.3) has the following meaning:

$$||u(t)||_{L^2(D)} \leqslant \frac{c}{\sqrt{t}} \qquad ||u(t) - u_\Gamma||_{H^2(\tilde{D}')} \leqslant \frac{c}{t^{1/4}} \tag{2.6}$$

$$||u(t)||_{L^2(\Gamma)} \leqslant \frac{c}{t^{1/4}} \tag{2.7}$$

where $\tilde{D}'$ is any compact strictly inner subdomain of D'. Here and below, $c > 0$ denote various positive constants independent of t or any other parameters which vary.

Estimates (2.6) and (2.7) are proved in [47–49]. It is proved in [50] that if $q_j(x)$, $j = 1, 2$, generate the scattering amplitudes $A_j(\alpha', \alpha, k)$ then the following relation holds:

$$-4\pi A(\alpha', \alpha, k) = \int_{R^3} p(x)u_1(x, \alpha, k)u_2(x, -\alpha', k)\,dx \tag{2.8}$$

where

$$A := A_1 - A_2 \qquad p := q_1 - q_2 \tag{2.9}$$

and u_j is the scattering solution corresponding to q_j. Formula (2.8) is derived in [50] under the assumption that $q_j(x) \in L^p_{\text{loc}}$, $p > n/2$ and $q(x)$ is in $L^1(B'_R)$, where $B'_R := R^3 \setminus B_R$, $B_R := \{x : |x| \leqslant R\}$, $R > 0$ is an arbitrary large fixed number.

In [51] an analogue of (2.8) is derived for obstacle scattering. Namely, it is proved in [51] that if Γ_j, $j = 1, 2$, are bounded sufficiently smooth (say, Lipschitz) surfaces and A_j are the corresponding scattering amplitudes, $A_j := A_{\Gamma_j}$, $A := A_1 - A_2$, then [51, formula (4)]

$$-4\pi A(\alpha', \alpha, k) = \int_{\Gamma_{12}} [\bar{u}_{1N}(s, \alpha, k)u_2(s, -\alpha', k) - u_1(s, \alpha, k)u_{2N}(s, -\alpha', k)]\,ds \tag{2.10}$$

where N is the exterior unit normal to $\Gamma_{12} = \partial D_{12}$, where $D_{12} := D_1 \bigcup D_2$.

(2) We claim that, uniformly in $t_1, t_2 \in [1, \infty]$, the following stability estimate holds

$$\sup_{\alpha',\alpha\in S^2; 0<k_1\leqslant k\leqslant k_2<\infty} |A_{D_1}^{(t_1)}(\alpha',\alpha,k) - A_{D_2}^{(t_2)}(\alpha',\alpha,k)| \leqslant c\{[\min(t_1,t_2)]^{-1/4} + \rho(D_1,D_2)\} \tag{2.11}$$

where $c =$ constant > 0, c is independent on $t_j \in [1, \infty]$ and on $D_j \subset B_a$, $j = 1, 2$, such that the boundaries of D_j satisfy the estimate $||\phi_j||_{C^{2,\nu}} \leqslant \Phi_\nu$.

The distance $\rho(D_1, D_2)$ in (2.11) is defined by the formula

$$\rho(D_1, D_2) := \sup_{x\in\partial D_1} \inf_{y\in\partial D_2} |x - y|.$$

(3) Note that if $t \in [1, t_0]$, where $1 < t_0 < \infty$ is any fixed number, then the following estimate can be derived from (2.8):

$$\begin{aligned} &\sup_{\alpha',\alpha\in S^2; 0<k_1\leqslant k\leqslant k_2<\infty} |A_{D_1}^{(t_1)}(\alpha',\alpha,k) - A_{D_2}^{(t_2)}(\alpha',\alpha,k)| \\ &\quad\leqslant \frac{c}{4\pi}|t_1 - t_2| \int_{D_1\bigcap D_2} \mathrm{d}x + \frac{ct_0}{4\pi}\int_{D_{12}\setminus(D_1\bigcap D_2)} \mathrm{d}x \\ &\quad\leqslant \frac{c}{4\pi}|t_1 - t_2| \left|D_1 \bigcap D_2\right| + \frac{ct_0}{4\pi}\{|\partial D_1| + |\partial D_2|\}\rho(D_1, D_2) \\ &\quad\leqslant c\{|t_1 - t_2| + \rho(D_1, D_2)\}. \end{aligned} \tag{2.12}$$

Here we have used the known estimate [50, 52]

$$\max_{x\in R^3; \alpha\in S^2; 0<k_1\leqslant k\leqslant k_2<\infty} |u_j| \leqslant c. \tag{2.13}$$

In (2.12), $|\partial D_j|$ denotes the area of the surface ∂D_j and $|D_1 \bigcap D_2|$ denotes the volume of $D_1 \bigcap D_2$.

(4) If $t_1 = t_2 = +\infty$, then the stability estimate

$$\sup_{\alpha',\alpha\in S^2; 0<k_1\leqslant k\leqslant k_2<\infty} |A_1(\alpha',\alpha,k) - A_2(\alpha',\alpha,k)| \leqslant c\rho(D_1, D_2) \tag{2.14}$$

follows from formula (2.10), since

$$\sup_{s\in\Gamma_j; \alpha\in S^2; 0<k_1\leqslant k\leqslant k_2<\infty} |u_{jN}(s,\alpha,k)| \leqslant c$$

$$\sup_{\alpha'\in S^2; s\in\Gamma_{j+1}; 0<k_1\leqslant k\leqslant k_2<\infty} |u_j(s,-\alpha',k)| \leqslant c\rho(D_1, D_2).$$

Here $\Gamma_3 := \Gamma_1$, $j = 1, 2$.

The basic result (2.11), which contains both stability estimates (2.12) and (2.14), is of interest because the inequality (2.11) holds *uniformly in* t, $t \in [1, \infty]$.

(5) As an example, we present here the results on the dependence $c(k)$ in (2.14) for the special case of the scattering potential. We claim that the constant c in (2.14) is of the order $O(k^2)$ as k goes to infinity under the following assumptions: (i) $J = 1$; (ii) $s \cdot N > b > 0$ for s in S_1 ($S_1 := \Gamma$) and for s in the perturbed surface, say S_2; here N is the outer normal to S_1 (or S_2) at the point s, $b > 0$ is a constant independent of s, k and other parameters.

Proof. If (ii) holds, then, from estimate (2.6) in [53, p 66], it follows that: $||v||_{B_R} < c$; c is always assumed to be independent of k; $v := u - u_0$, where u is the scattering solution corresponding to S_1; and u_0 is the plane wave. From this and the Helmholtz equation, one obtains $\|v\|_2 < ck^2$, where $\|v\|_2$ is the Sobolev-space H^2 norm. Let $|v_N|$ stay for the $L^2(S_1)$ norm of v_N on S_1. Then, an interpolation inequality yields the desired estimate: $|v_N| < ck^{3/2}$. This estimate implies the claim that the constant c in (2.14) is of the order $O(k^2)$ as k grows to infinity. Indeed, estimating integrals in (2.10) by Cauchy's inequality, one obtains the product sum of the terms of type $|v_N|\ |v|$ and terms of lower order in k which are easy to estimate by $O(k^{3/2})$. By an interpolation inequality, the norm $|v|$ is $O(k^{1/2})$, so the result follows. Let us formulate the known interpolation inequalities used above (see [40])

$$\|D^r v\|_{L^2(S_1)} < ct^{3/2-r}\|v\|_2 + t^{-1/2-r}\|v\| \tag{2.15}$$

where $\|v\|$ is the L^2 norm in $B_a \setminus D$, $\partial D = S_1$, $t > 0$ in (2.15) is an arbitrary parameter, and $r = 0$ or 1. Take $r = 0$ in (2.15) and minimize the right-hand side of (2.15) in $t > 0$, using the formulae $\|v\|_2 < ck^2$ and $\|v\| < c$, to obtain the estimate $O(k^{1/2})$ for the right-hand side. A similar argument for $r = 1$ yields the estimate $O(k^{3/2})$ as claimed. □

Remark. The order in k as $k \to \infty$ in the estimate for the constant c in (2.14) is not optimal. The optimal order is, probably, O(1). For a ball, for instance, we can prove that $|v_N| = O(k)$ rather than $O(k^{3/2})$ and $|v| = O(1)$ rather than $O(k^{1/2})$. This yields $c = O(k)$ as $k \to \infty$. The estimate based on the Cauchy inequality, used in our derivation, does not take into account possible cancellations during integration in (2.10) due to oscillations of the integrand for large k. The optimal orders are: (i) O(1) for $|v|$; (ii) $O(k)$ for $|v_N|$ and; (iii) O(1) for the cross section as $k \to \infty$. These conclusions can also be obtained from the geometrical optics approximation (see formula (150.16) in [54])

(6) Let us formulate the result proved in [49].

Theorem 1. Under the assumption made in section 2.1, estimate (2.11) holds with the constant $c > 0$ independent of t, where $t \in [1, \infty]$, $D_j \subset B_a$, $\partial D_j \subset C^{2,\nu}$ and $\|\phi_j\|_{C^{2,\nu}} \leqslant \Phi_\nu$.

In section 3, the proof of estimate (2.14) is given for the case $t_1 = t_2 = \infty$, which is of interest in applications. In section 4, applications are discussed.

3. Proof of the stability estimate (2.14)

Let us assume that

$$q_j(x) = \begin{cases} +\infty & \text{in } D_j \\ 0 & \text{in } D_j' := R^n \setminus D_j,\ n \leqslant 2. \end{cases} \tag{3.1}$$

This is the case discussed in section 2.4 (see formula (2.14)). We assume $n = 3$ for definiteness. The argument is the same for $n \geqslant 2$.

There are three ways to prove estimate (2.14) under assumption (3.1). One way is to take $t_1 = t_2 = +\infty$ in (2.11) and note that the right-hand side equals $c\rho(D_1, D_2)$ if $t_1 = t_2 = +\infty$. The second way is to take $t_1 = t_2 = t < \infty$ and then let $t \to +\infty$ and use formula (2.8) and estimates (2.6) and (2.7). These estimates allow one to derive formula

(2.10) from which estimate (2.14) follows. Estimate (2.14) is a particular form of estimate (11) for the case when $\min(t_1, t_2) = +\infty$. The third way is based on estimate (2.10). Let us use this way. We assume that the distance $\rho(D_j, D_m)$, $j \neq m$, is much greater than the distance $\rho(D_j, \tilde{D}_j)$, where $\tilde{D}_j$ is the perturbed domain D_j. The number J of the connected components of the domain D is fixed and finite. Therefore, the input of the variation of ∂D in the scattering amplitude is of the order of magnitude of the input of the variation of ∂D_j, $1 \leqslant j \leqslant J$. Therefore, one may use formula (10) assuming that ∂D has one connected component ∂D_1 and $\partial D_2 := \partial \tilde{D}_1$ is a small variation of ∂D_1 in the sense that $\rho(D_1, D_2)$ is small. It follows from (2.10) that

$$|A(\alpha', \alpha, k)| \leqslant \frac{1}{4\pi} \int_{\Gamma_1'} |u_{1N}(s, \alpha, k) u_2(s, -\alpha', k)| \,\mathrm{d}s + \int_{\Gamma_2'} |u_1(s, \alpha, k) u_{2N}(s, -\alpha', k)| \,\mathrm{d}s := I_1 + I_2 \tag{3.2}$$

where Γ_1' is the part of Γ_1 which lies outside D_2 and Γ_2' is the part of Γ_2 which lies outside D_1.

One can use the following estimates:

$$\gamma := \max_{j=1,2} \sup_{s \in \Gamma_j; \beta \in S^2; 0 < k_1 \leqslant k \leqslant k_2 < \infty} |u_{jN}(s, \beta, k)| \leqslant c \tag{3.3}$$

$$\max_{j=1,2} \sup_{s \in \Gamma_j'; \beta \in S^2; 0 < k_1 \leqslant k \leqslant k_2 < \infty} |u_{j+1}(s, \beta, k)| \leqslant c\rho(D_1, D_2) \qquad u_3 := u_1 \tag{3.4}$$

and formula (3.2) to obtain the desired estimate (2.14). Let us discuss estimates (3.3) and (3.4). The constant c in (3.3) and (3.4) depends on the parameters k_1, k_2, a and on the parameter Φ_ν, which is introduced in section 2.1 and which describes the smoothness of the boundary: $\|\phi_j\|_{C^{2,\nu}} \leqslant \Phi_\nu$. This constant does not depend on the particular choice of D_j. Let us prove the last claim. Suppose, on the contrary, that there exists a sequence D_{jn} of the obstacles $D_{jn} \subset B_a$, $\|\phi_{jn}\|_{C^{2,\nu}} \leqslant \Phi_\nu$, such that $\gamma_n \geqslant c_n$, $c_n \to \infty$, where c_n are the constants in (3.3) and (3.4) and γ_n is γ for the obstacle D_{jn}, $n = 1, 2, \ldots$. By the Arzela–Ascoli compactness theorem, one can assume that

$$\phi_{jn} \xrightarrow{C^{2,\nu'}} \psi_j \qquad 0 < \nu' < \nu \qquad u_{jn} \xrightarrow{H^2_{\mathrm{loc}}} u_j \qquad n \to \infty$$

where u_j is the scattering solution corresponding to the limiting configuration of the surfaces Γ_1 and Γ_2. For fixed surfaces Γ_1 and Γ_2, estimates (3.3) and (3.4) hold [53].

Note that it is sufficient to prove estimate (3.3). Indeed,

$$|u_1(s, \beta, k)| = |u_1(s, \beta, k) - u_1(\tilde{s}, \beta, k)| \leqslant \sup |u_{1N}(s, \beta, k)| |s - \tilde{s}| \leqslant c\rho(D_1, D_2)$$

where $s \in \Gamma_2'$, $\tilde{s} \in \Gamma_1$, $u_1(\tilde{s}, \beta, k) = 0$ and the segment $\tilde{s}s$ is directed along the normal to Γ_2'. A similar argument is valid for $u_2(s, \beta, k) = 0$, $s \in \Gamma_1'$.

If $\Gamma_{jn} \to \Gamma_j$ in the sense $\phi_{jn} \xrightarrow{C^{2,\nu'}} \psi_j$ as $n \to \infty$, then $u_{jNn} \to u_{jN}$ as $n \to \infty$ (uniformly in $s \in \Gamma_j$ and in the parameters $\beta \in S^2$, $k \in [k_1, k_2]$, $0 < k_1 < k_2 < \infty$) so that $\gamma_n \to \gamma$ as $n \to \infty$. Here, γ is the number defined by the left-hand side of (3.3) with u_j corresponding to the limiting surfaces Γ_j. Since this $\gamma < \infty$, one obtains a contradiction: the inequality $\gamma_n \geqslant c_n \to +\infty$ contradicts the equation $\gamma_n \to \gamma < \infty$. This contradiction proves that the constant c in (3.3) and (3.4) does not depend on the particular choice of the obstacles D_j as long as the two conditions are satisfied: $D_j \subset B_a$, $\|\phi_j\|_{C^{2,\nu}} \leqslant \Phi_\nu$, and the parameters a, Φ_ν, k_1 and k_2 define the value of c in (3.3), (3.4) and (2.14).

4. Conclusions

In connection with the problem of correlation effects in QCS and in mesoscopic systems, the consideration presented in sections 2 and 3 are of considerable interest. In particular, the estimate for the scattering amplitudes given by formula (2.14) is valid for the general case of singular potentials $q(x)$ supported in a compact region D. In this case, the corresponding classical repeller Ω_R is, in general, chaotic. So, result (2.14) means that the strong quantum correlations in the scattering amplitudes exist in some region of parameters, even for classically chaotic (irregular) scattering and are of the universal nature. The latter means that the quantum correlations in this region of parameters do not depend on the specific character of the resonance structure of the scattering amplitude. Estimate (2.14) includes the constant c which actually depends on the systems parameters

$$c = c(k_1, k_2, a, \Phi_\nu). \tag{4.1}$$

This is why it is difficult to establish a direct relation between the region of parameters where estimate (2.14) is valid and the region ($\delta E > \Gamma > \Delta E$) where the Ericson fluctuations are important.

The analytical and experimental investigations of the dependence (4.1) are of considerable interest for the further development of our understanding of the correlation effects in the processes of QCS. The function (4.1) can be investigated, for example, in resonant tunnelling experiments in mesoscopic systems when the samples are prepared with small boundary variation (scattering potential). Another possibility to investigate the correlation and fluctuation effects in QCS can be realized by the microwave experiments (see, for example, [55]). The main idea, which is used in these experiments, is that the Schrödinger equation for a free particle reduces to the Helmholtz equation which describes the propagation of classical waves. This correspondence was utilized in [55] to investigate the role of fluctuations in CS. In our opinion, this method is rather promising: it allows one to imitate the ballistic regime in mesoscopic systems, taking into account scattering, and to study the correlation effects in mesoscopic systems using a microwave technique.

Acknowledgments

AGR thanks LANL for support. GPB thanks Don Cohen, Gary Doolen and J Mac Hyman of the Center for Nonlinear Studies, Los Alamos National Laboratory for their hospitality. GPB also gratefully acknowledges the partial support of the Linkage Grant 93-1602 from the NATO Special Programme Panel on Nanotechnology.

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